The Measure of a «Pragmatic Information» In a Probabilistic Environment : A Review of Whittemore-Yovits' Model. (1)

ABSTRACT

This paper provides a summary of Whittemore-Yovits' model. Some objections have been raised, mainly the use of the notion of distance in the computation of the pragmatic information. Instead of the distance we are suggesting the use of the probability distribution to frame the decision outcome. The pragmatic information will rather be the diversity of probability relative to the set of choices.

RESUME

Cet article présente l'éssentiel du modèle Whittemore-Yovits concernant le calcul de l'information pragmatique en milieu probabiliste. L'usage de la notion de distance dans le calcul de l'information pragmatique a été remise en question. Il a été suggéré plutôt l'usage d'une distribution de probabilité comme cadre prévisionnelle du choix et du résultat d'une décision. L'information pragmatique sera dans ces conditions la distribution de probabilité relative aux choix décisionnelles.

THE MEASURE OF A "PRAGMATIC INFORMATION" IN A PROBABILISTIC ENVIRONNEMENT: A REVIEW OF WHITTEMORE-YOVITS' MODEL(1)

by Victor J. Bikaï-Nyunaï, Ph.D.
Associate Professor at ESSTIC
The University of Yaounde II

1. SUMMARY OF WHITTEMORE-YOVHTS MODEL

1.1 Hypotheses Statement

Whittmore-Yovits' model is rooted in three fundamental hypotheses :

a. «Information» and «decision-making» have a constant relationship. It is implicit in the model that this relationship has a causal nature.

b. Information «exists», therefore it can be objectively measured, and the means to do so are also available to the decision-maker, because they «exist».

c. The Decision-Maker must have somehow a decision making tool (a model) allowing him/her to evaluate the discrepancy between predicted and actual «observables» as a result of any decision for which he/she is accountable. The rationale of this is that the decision-maker should have a feedback that allows him/her to correct any deviation from prediction.

1.2 Assumptions

Seven major assumptions underlie the structural basis of Whittemore-Yovits model.

Assumption 1

The decision maker is confronted with a set of alternative decisions that determine his/her action. That set is noted :

$A=\{a_1, a_2, \ldots, a_i, \ldots, a_m\}$
$i=\{1,2,\ldots,m\}$

Assumption 2

Each individual alternative a_i is determined to produce a certain level of output o_j. The possible outputs constitute a set noted :

$O=\{o_1, o_2, \ldots o_j, \ldots, o_n\}$
$j=\{1,2,\ldots,n\}$

Assumption 3

Based on the goal structure of the organization, the outcome o_j may be given a relative value $V(o_j)$

Assumption 4

Each element of decision, that is each ordered pair (a_i, o_j) is contingent upon a state of nature s_k. The authors defined that state of nature as one's understanding, the rôle of the decision maker, his/her rationality, the environment and/or a combination of these elements. The set of states of nature is noted :

$s_k=\{s_1, s_2, \ldots, s_k, \ldots, s_r\}$
$k=\{1,2,\ldots,r\}$
$r= nxm$

Assumption 5

The Probability that a couple (a_i, o_j) or the event (a_i, o_j) occur under the influence of the state of nature s_k is noted $P(s_k)$, with the condition that :

$$\sum_{k=1}^{r} P(S_k) = 1$$

Assumption 6
The probability that an alternative decision a_i produces an outcome o_j is w_{ij} and it is noted :

$$p(a_i, o_j) = w_{ij}$$

With

$$\sum_{j=1}^{n} w_{ij} = 1$$

Assumption 7
The new information that is used by the decision-maker has the original effect to change the various representatives of his/her uncertainty. And that change will be noted :

$$\left[w_{ij}^{k}\right]_{t+1} = \left[w_{ij}^{k}\right]_{t} + \Delta w_{ij}^{k}$$

$$\left[P(s_k)\right]_{t+1} = \left[P(s_k)\right]_{t} + \Delta P(s_k)$$

$$\left[\upsilon(o_j)\right]_{t+1} = \left[\upsilon(o_j)\right]_{t} + \Delta \upsilon(o_j)$$

1.3 The Major Findings

Finding 1
The concept of executional uncertainty is one of the major claims of the authors. They contend that the decision-maker must overcome that uncertainty in order to choose the appropriate alternative decision a_i that yields a desired outcome o_j.

Finding 2
The pragmatic information I(D) contained in a set of data or message is equal to the difference of the value of the decision-state of the decision-maker after and before receipt of the message. In mathematical form, this statement will be noted :

$$I(D) = V(DS_{t+1}) - V(DS_t)$$

2. MEANING OF THE RESULTS

a) Four functions can be attributed to the generalized information system; these include : the information acquisition and dissemination function, the decision making function, the execution function, and the transformation function.

b) Each of these functions is committed in collecting, storing, operating, and disseminating information.

c) Within the context of the generalized system, information is what is used as raw material, as resource to generate observables, or economic outputs.

d) The decision-maker may be said to «transduce» information to observables. (P. 233).

3. DISCUSSION

The fourth meaning in line (d) may be debatable. It would suggest that if a decision-maker has a model of achieving his/her optimum for instance the one shown in Fig 1, his/her predecided output $A(X)+A_0$ will be achieved exactly as observables.

Fig. 1

$$F(X) = A(X) + A_0$$

F = Function
X = Choices
A = Matrix of Choices
M_0 = Parameter

This can be true in extreme cases, but most of the predicted outputs will be different from the observable achievements. This is in fact the foundation of the decision-making role, to receive «feed back» (P.233) for future readjustments. If predictions are equal to achievements in principle, the decision-making function may be seen as redundant.

Unless there is a typesetting error, the execution function in this model is what might be called the operational function, that is the function that transforms inputs into outputs or observables it cannot be otherwise, given the direction of the iteration Fig. 2.

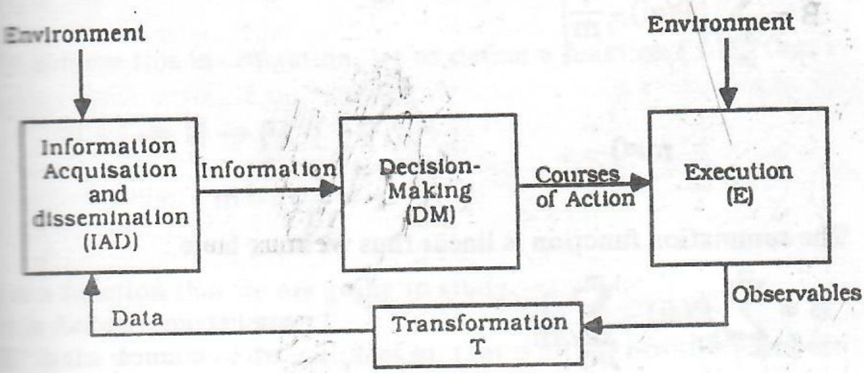


Fig. 2 The generalized information system model.

Most importantly, it is in the execution level that either the value-added is gained or the lost is inflicted to the operation as compared to the actual level of observable outputs. At this stage tremendous changes have to be operated in Whittemore and Yovits' model .

The nature of Ackoff's factor in Whittemore and Yovits model.

$$B = \sum_{i=1}^{m} \left| P(ai) - \frac{1}{m} \right| \qquad m \neq 0$$

will shift from «distance» (P. 228) to a relative entity belonging to the set of real numbers.

As a distance, B, belongs to N, the set of natural numbers therefore excludes all the negative ones. But we have advocated that the execution function may or may not meet the expectations of the decision-maker, thus B is susceptible to be positive for a value-added or negative for a lost. In this context, if B may take values from $-\infty$ to $+\infty$ the domain of validity of B will be R (relative numbers). The value of B is therefore as following :

$$B = \sum_{i=1}^{m} \left(P(ai) - \frac{1}{m} \right)$$

$$m \neq 0$$

The summation function is linear thus we must have :

$$B = \sum_{i=1}^{m} P(ai) - \sum_{i=1}^{m} \frac{1}{m}$$

$$B = \sum_{i=1}^{m} P(ai) - \frac{1}{m}$$

$\sum_{i=1}^{m} P(ai)$ is the probability of occurance of the universe of all the possible decisions, thus

$$\sum_{i=1}^{m} P(ai) = 1$$

And by substitution, the system (4) becomes as follows :

$$B = 1 - \frac{1}{m} \qquad m \neq 0$$

B is the potential observed gain or lost subsequent to the decision in information-rich environment. Our next problem is to investigate what is the probabilistic law that rules the gain and the lost. Represented by :

$B = -\frac{1}{m} + 1$, with $m \in \Re^{*+}$.

To achieve this investigation, let us define a function f such that :

$$f : \Re -]-\infty,1] \rightarrow [0,1[$$

$$m \mapsto -\frac{1}{m} + 1$$

f is a function that we are going to study.
m is Ackoff's parameter.
R^{*+} is the domain of definition of m, that is all the positive numbers excluding zero.

Condition of First Order.

The derivative of f follows the rule :

$$\left(\frac{u}{\upsilon}\right)' = \frac{\upsilon u' - u\upsilon'}{\upsilon^2}$$

$$f(m) = -\frac{1}{m} + 1$$

$$= \frac{m-1}{m} \quad \text{, if } u = m - 1$$

$$\upsilon = m$$

$$f'(m) = \frac{m(m-1)' - (m-1)(m)'}{m^2}$$

$$= \frac{mx1 - m+1}{m^2}$$

$$= \frac{1}{m^2}$$

$$f'(m) = \frac{1}{m^2} \quad \text{always positive}$$

Variation of f(m)

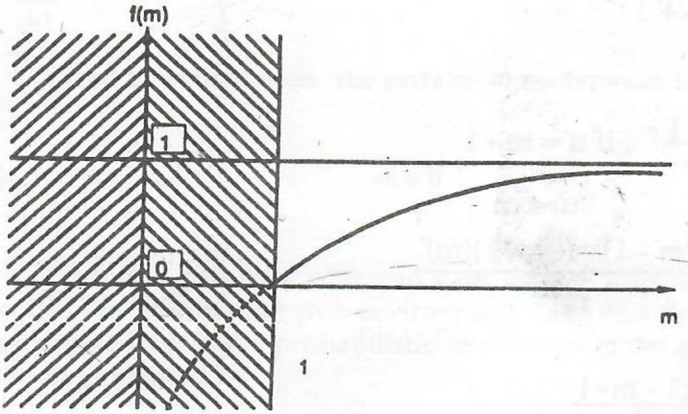
$$\lim_{m \to 0+} f(m) = -\infty \qquad \lim_{m \to +\infty} f(m) = 1$$

$$f(m) = 0 \quad m = 1$$

in summary, we can put these mathematical findings into a single table.

m	0		1		$+\infty$
f'(m)	‖		+		
f(m)	‖	$-\infty$	0	↗	1

Graphic Representation of f(m).



Interpretation.

1. The shaded area represents the excluded values of m and f(m).

2. m is the number of alternative decisions the decision-maker is confronted with.

3. The analysis shows theoretically that if m is between O+ and 1 f(m) will be negative. Practically it means that in an either-or environment or at the extreme when the decision-maker does not have choices at all but one single dictorial decision, the probability that it be wrong and incur a loss to an organzation is evident. The value of the function f (m) goes below the horizontal axis. That region below the horizontal axis corresponds to negative values of f(m) in theory. But it does not make sense to have negative probabilities. That is why the probability curve is dotted.

4. As m increases, f(m) increases slowly in a logarithmic way up to the asymptote f(m)=1. What this means is that as the set of choices grows, the decision-maker have better chances to have a positive value-added. The model shows that the more the decision-maker has many alternatives, the better the chance of gain. But the magnitude of the certainty of gain is limited by the structure of the decision making. That structure is represented in the model by the asymptote at f(m)=1. This asymptote represents the probability of certainty. It has just been demonstrated that the distance that separates the decision-maker from the right decision is not the appropriate tool to garantee a successful decision. Rather, it is the number of alternatives that determines the probability of gain. The argument against the use of the notion of distance comes up with the computation of the value of the decision state DS at time t. (P.229)

$$V(DS_t) = \frac{\sum_{i=1}^{m} \left| P_t(ai) - \frac{1}{m} \right|}{2 - \frac{2}{m}}$$

$$\text{if } t \rightarrow +\infty, \sum_{i=1}^{m} \left| P_t(ai) - \frac{1}{m} \right| \rightarrow \sum_{i=1}^{m} P_t(ai) - \frac{1}{m}$$

We know that $\sum_{i=1}^{m} P_t(ai) \to 1$

$t \to +\infty$

Therefore

if $t \to +\infty$

$$V(DS_t) \to \frac{1-\frac{1}{m}}{2-\frac{2}{m}} = \frac{1}{2}\left[\frac{1-\frac{1}{m}}{1-\frac{1}{m}}\right] = \frac{1}{2}$$

$t \to +\infty$

$$V(DS_t) \to \frac{1}{2}$$

What means 1/2 in the scale of value of Whittemore-Yovits' model? We probably do not know.

Therefore the finding of Whittemore and Yovits' model is more theoretical than it is practical.

If $V(DS_t)$ does not have a practical meaning, neither $V(DS_{t+1})$ nor I(D) will have one.

This puts into question some aspects of the model.

4. ALTERNATIVE

We should not be interested in the distance that separates the DM from the state of undeterminism. In a universe of probability everything can occur as long as the probability of occurrence is different from O. By taking the best position that minimizes the distance, the DM can still engage a catastrophic action. This proves that the distance in this context is not an appropriate leverage to prevent a decision making disaster.

However the disaster or a bad result can still be prevented even if the distance is maximum. As long as that result is governed by a probabilistic law. Therefore absolute value, should be substituted with relative values. The relative values will show the actual variation of the output subsequent to a decision. The output could be a gain (positive output), or a loss (negative output) depending on the value of B, (or expression 5).

The graphic representation of f(m) when a decision maker has more than one alternative decision, or $m \in \{1,+\infty\}$ shows that the probability for him/her to have a gain varies from 0 to 1, that is from uncertainty to certainty. Because $m \in \{1,+\infty\}$, Whittemore and Yovits' model is an open system model of decision making.

CONCLUSION

The model that is suggested here for re-examination is among the most sophisticated in the field. Although very theoretical, the relevance of the issue makes the model an interesting subject of discussion. This paper argues that the assessment of the distance that separates the decision-maker from the the state of uncertainty is not an accurate way of overcoming uncertainty. We are suggesting that a decision-maker should rather have a distribution of probability of gain or loss, that is, a set of stochastic alternatives. We have shown that under a set of stochastic alternative decisions, the probability of gain will vary from O to l, that is, from uncertainty to certainty.

(1) Bruce J. Whittemore and M.C. Yovits, A Generalized Conceptual Development for the Analysis and Flow of Information : Journal of the American Society for Information Science. (May-June 1973) : 221-231.